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Development of Indian Mathematics in Ancient Periods

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Abstract:

History of Indian mathematics has a glorious history over 5000 years. The basic mathematics in Indian subcontinent comes from the Indus Valley Civilization. In Vedic period Sulbasutras have developed for different geometrical constructions. This geometrical construction has developed with the demand of religious needs. The present paper tries to discuss different phases of development of the Indian mathematics during ancient period.

Keywords: Indus Valley Civilization, Sulbasutras, Geometry, Indian Mathematics, Ancient Period

Introduction:

The history mathematics shows that the great works of Indian Mathematicians happened in ancient period. The oldest branch of Indian Mathematics is geometry. The basic of whole Indian ancient Mathematics is geometry. In ancient India, algebra is also having leaned on geometry. Different arithmetic operations i.e., addition, subtraction, multiplication and division were demonstrated geometrically in ancient Indian Mathematics [2][5][6].

The basic mathematics in Indian subcontinent comes from the Indus Valley Civilization. Indus Valley Civilization excavated at Harappa and Mohenjo-daro also indicates the knowledge of geometry [1][7].

Vedic period was golden age of ancient Indian mathematics. In this period the Sulbasutras were developed mainly for religious needs. The Sulbasutras are manuals of geometrical constructions [4].

Different Phases of Ancient Indian Mathematics:

Indus Valley Civilization:

In Indian subcontinent the oldest civilization is Harappan Civilization which is known to have consisted of two large cities, Harappa (in the Punjab region) and Mohenjodara (near the Indus River in the Sindh region). The exact date of initiation of the civilization remains confusing till date. According to different papers published on Indus Valley Civilization it is assumed that that this civilization is 8000years old. The well-planned cities in Indus Valley Civilization show the practical knowledge of geometry [1][2].

Contribution in the field of Mathematics during Indus Valley Civilization:

(i) Harappans used weights in the ratios 0.05, 0.1, 0.2, 0.5, 1.2, 5, 10, 20, 50, 100, 200 and 500[1][2].

(ii) In Valley Civilization bricks used were in the ratio 4:2:1, which is best for effective adherence between the bricks [1].

(iii) In Harappan culture, it is to be found the practical knowledge of geometry like concentric and intersecting circles and triangles. In that period weights in different geometrical shapes like cone, cylinder and cuboid were found[1][2][4].

Vedic Period (1500-500 BC):

After Indus Valley Civilization, the development of Indian Mathematics can be found in the Vedas. The Vedas were mainly containing the religious texts. There are four Vedas:

(i) *Rigveda*: It contains hymns about their mythology

(ii) *Samaveda*: It consists of hymns about religious rituals

(iii) *Yajurveda*: It contains instructions for religious rituals

(iv) *Atharvaveda*: It consists of spells against enemies, sorcerers and diseases.

In 1981**,** Sri Bharti Krishna Tirthaji, rediscovered the Vedic Mathematics from the Vedas, which contains sixteen sutras and thirteen sub-sutras [3][9]. The sixteen sutras are:

(1) *Ekadhikena Purvena*: By one more than the previous one.

(2) *Nikhilam Navatascaramam Dasatah*: All from 9 and last from 10

(3) *Urdhva-tiryakbhyam*: Vertically and crosswise.

(4) *Paravartya Yojayet*: Transpose and adjust.

(5) *Sunyam Samyasamuccaye*: When the samuchayas are same, then it is Zero

(6) *(Anurupye) Sunyamanyat*: If one is in ratio, the other is zero.

(7) *Sankalana-vyavakalanabhyam*: By addition and by subtraction.

(8) *Puranapuranabhyam*: By the completion and non-completion.

(9) *Calana-kalanabhyam*: Differences and similarities.

(10) *Yavadunam*: Whatever the extent of its deficiency.

(11) *Vyastisamastih*: Part and Whole.

(12) *SesanyankenaCaramena*: The remainders by the last digit.

(13) *Sopantyadvayamantyam*: The ultimate and twice the penultima.

(14) *Ekanyunena Purvena*: By one less than the previous one.

(15) *Gunitasamuccayah*: The product of sum is equal to sum of the product.

(16) *Gunakasamuccayah*: Factors of the sum is equal to the sum of factors.

Contribution in the field of Mathematics during Early Vedic Period:

(i) Decimal number system.

(ii) Arithmetical operations i.e., addition, subtraction, multiplication and division.

(iii) Fraction, Squares, Cubes, Square roots, cube roots.

In later part of Vedic period, Sulbasutras were developed for geometrical construction. In Vedic period different geometrical constructions developed with the demand of religious needs.There are nine extant Sulbasutras of which four, Baudhayane, Apastanba, Manava and Katyayana Sulbasutras are of significance from a mathematical point of view[4][5][8].

To Construct right angled triangle in Vedic Period the ratio 3:4:5 was used [5].

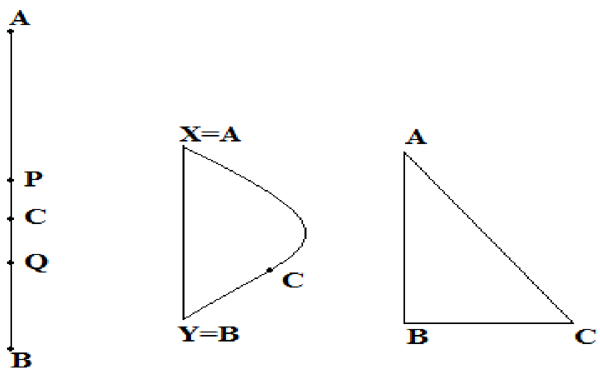

Figure 1: Construction of right angled triangle

To construct the Mahavedi different triples were used viz (3, 4, 5) and (5, 12,13); (15, 20, 25) and (12, 16, 20); (5, 12, 13) and (15, 36,39); (8, 15, 17) and (12, 35, 37)etc[5][10].

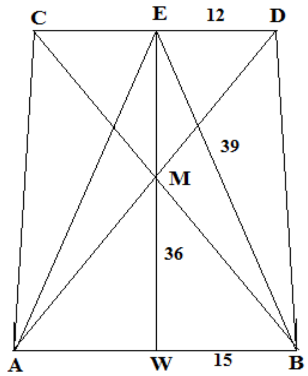

Figure 2: The *Mahavedi*

In Apastamba, Baudhayana and Katyayana, different rules were given

- (i) To draw a square equal to the difference of two squares
- (ii) To convert a rectangle into a square
- (iii) To convert a square into a rectangle
- (iv) To convert a rectangle or square into a trapezium with the shorter parallel side given
- (v) To convert a Trapezium into an equivalent rectangle or square.

Contribution in the field of Mathematics during later Vedic Period:

- (i) Astronomy
- (ii) Geometry
- (iii) Pythagorean triples
- (iv) Irrational numbers

The Jaina Mathematics (400 BC – 200 AD):

In the history of Indian Mathematics, Jaina Mathematics is considered as a bridge between ancient and classical periods. Vedic Mathematics was developed by religious needs, but in the period of Jaina, Mathematics was developed as a discipline. Jaina mathematicians were working in the field of large numbers, they divided the large numbers into three categories: enumerable, innumerable and infinite. A large number found in Jaina book of cosmology [1], which was used to tell the age of the universe in years. The number is:

$2^{588} =$

1013 065324 433836 171511 818326 096474 890383 898005 918563 696288 002277 756507 034036 354527 929615 978746 8515512 277392 062160 962106 733983 191180 520452 956027 069051 297354 415786 421338 721071 661056[1][2].

Contribution in the field of Mathematics during Jainas period:

(i) Number Theory
(ii) Arithmetical operations i.e., addition, subtraction, multiplication and division.
(iii) Law indices:$a^m X a^n = a^{m+n}$
(iv) Geometry of circles
(v) Theory of permutations & combinations
(vi) Value of π approximation to $\sqrt{10}$

The Baksali Manuscript:

The Baksali Manuscript [6] was discovered by a farmer at a village at Baksali near Peshwar in 1881 AD. The manuscript was written by Gatha language on birch bark. The date of Baksali manuscript is controversial. The manuscript contains the following problems:

(i) The problems related to arithmetic and algebra
(ii) Few isolated problems of Geometry
(iii) Methodfor calculating square root like $\sqrt{41}, \sqrt{105}, \sqrt{889} \& \sqrt{339009}$
(iv) Sum of the Series (Arithmetic Progrssion)
(v) Problem involving indeterminate equations.

Contribution of Indian Mathematicians:
Aryabhata (476-550 AD):

Aryabhata was born in 476 AD at Pataliputra. At the age of 23 the great Indian Mathematician, Aryabhata wrote his famous book *Aryabhatiya*[2], which deals on astronomy .Aryabhata was perhaps the first to say that the earth rotates round its axis and gave the general integral solution of linear Diophantine equation $ax+by=c$[2].

Aryabhata's other Mathematical contributions include:

(i) Introduce the concept of 'Bijganita' (Algebra)
(ii) Discovery of trigonometric sine
(iii) Accurate evaluation of π
(iv) Perfection of decimal system of numbers
(v) Evaluation of AP and GP series
(vi) Square and cube root of numbers
(vii) Accurate calculation of number of days in solar year[1][2]

Varahamihira (505-587 AD)

Varahamihira's mathematical discoveries include :

(i) Place-value number system
(ii) Trigonometric formulae & sine tables

(iii) Problem of computing n_{C_r}[6]

Brahmagupta (598-660 AD):

Brahmagupta was born in 598 ADat Ujjain. Brahmagupta wrote the astronomical treatise: *Brahmasphutasiddhanta* (628 AD)[2] and *Khandakhadaka* (664 AD)[2]. In his *Brahmaphutasiddhanta,* he extensively dealt with the properties of cyclic quadrilateral and trapezium and the relation between their sides, diagonals and area. *Khandakhadaka* stated a new method to find out the intermediate sine values from the Aryabhata's sine table values [1][2].

Brahmagupta's other Mathematical contributions include:

(i) Arithmetic operation on 0
(ii) Computing square roots
(iii) Methods to solve indeterminate equations of the form $ax+c = by$
(iv) Solves quadratic indeterminate equations of the type $ax^2 + c = y^2$&$ax^2 - c = y^2$
(v) Sum of the series :

$$1^2 + 2^2 + 3^2 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

Mahavira (800-870 AD):

Mahavira's mathematical discoveries include:

(i) Arithmetical operations
(ii) Methods to decompose integers and fractions into unit fractions
(iii)Methods of squaring numbers
(iv)Operations relating to the calculations of areas
(v) Operations relating to shadows
(vi)Integer solutions of first degree indeterminate equation[1][2]

Sridhara (870-930 AD):

Sridhara gave the well known rule for solving quadratic equations popularly known as Sridharacharya's formulae [6].

If the quadratic equation is$ax^2 + bx + c = 0$, then his steps to solve it as:

Step I: $4a^2x^2 + 4abx + 4ac = 0$
Step II: $(2ax + b)^2 + 4ac = b^2$
Step III: $2ax + b = \pm\sqrt{b^2 - 4ac}$
Step IV: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Sridhara's other mathematical contributions include:

(i) Solving the problems involving ratios, simple interest, mixtures, purchase and sale, rates of travel, wages, and filling of cisterns
(ii) Problem of calculating n_{C_r}
(iii) Arithmetic and geometric progressions
(iv) Formulae for the sum of certain finite series are given
(v) Method for finding rational solutions of $Nx^2 \pm 1 = y^2, 1 - Nx^2 = y^2$

Bhaskaracharya (1114-1185 AD):

Bhaskarachary was born in Vijayapura and he was the last astronomer-mathematician of repute produced by ancient India. Bhaskaracharya's work *Sidhantasiromoni*[1][2] was divided into four parts:

(i) *Lilavati*: rule of arithmetic
(ii) *Bijaganita*: on algebra, root extraction
(iii) *Grahaganitadhya*: motion of planets
(iv) *Golodhyaya*: on calculation of sphere

Bhaskarachary's other Mathematical contributions include:

(i) Division by zero gives infinity
(ii) Solution of quadratic, cubic and quartic indeterminate forms
(iii) Proof of Pythagoras theorem
(iv) Introduced differential calculus
(v) Introduced spherical trigonometry [1][2]

Conclusion:

In ancient India, mathematical works were orally transmitted until approximately 500 BC; thereafter, they were transmitted both orally and in manuscript form. So, some ancient Indian mathematical works have been lost in the passage of time. Development of mathematics has contributed in the development of culture and civilization. In Vedic period, the Indian mathematics was the outcome of the religious needs. But in the Jaina period mathematics started to flourish as a discipline.

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